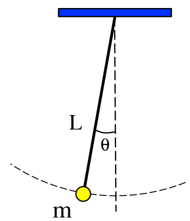
Problem 15.31

The simple pendulum system is shown to the right.



a.) What is the bob's *maximum speed*?

From our evaluation of simple harmonic motion, we know that:

$$v_{max} = \omega A$$

where "ω" is the bob's *angular velocity* and "A" its amplitude in meters.

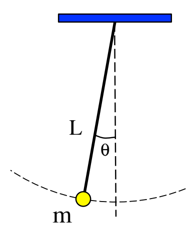
To get "A:"

$$A = R\theta$$
$$= (1.00\text{ m})\left[(15^\circ)\frac{\pi}{180^\circ}\right]$$
$$= .262\text{ m}$$

To get "ω:"

$$\omega = \left(\frac{g}{L}\right)^{1/2}$$
$$= \left(\frac{(9.80\text{ m/s}^2)}{(1.00\text{ m})}\right)^{1/2}$$
$$= 3.13\text{ rad/s}$$

a.) (con't.) What is the bob's *maximum speed*?



$$v_{max} = (3.13\text{ rad/s})(.262\text{ m})$$
$$= .820\text{ m/s} \quad \text{acting at equilibrium}$$

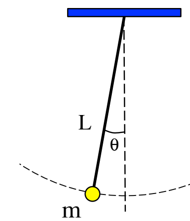
b.) What is the *maximum acceleration* magnitude?

$$a_{max} = \omega^2 A$$
$$= (3.13\text{ rad/s})^2(.262\text{ m})$$
$$= 2.57\text{ m/s}^2 \quad \text{acting at the extremes}$$

c.) What is the *maximum restoring force* on the bob?

$$\left|\vec{F}_{max}\right| = m\left|\vec{a}_{max}\right|$$
$$= (.250\text{ kg})(2.57\text{ m/s}^2)$$
$$= .643\text{ N}$$

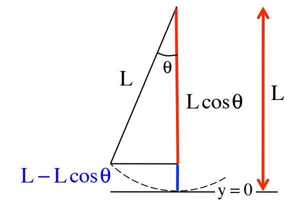
d.) Calculate everything another way.



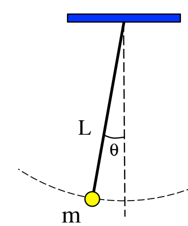
For v_{max}:

Taking the bottom of the arc as the y = 0 point and starting at the top of the swing and going to the bottom of the arc (note how "y" is defined to the right, where I've exaggerated the picture for viewing ease), *conservation of energy* suggests:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$
$$0 \quad + (mgy) + \quad 0 \quad = \frac{1}{2}m(v_{max})^2 + \; 0$$
$$\Rightarrow \quad +(mg(L - L\cos\theta)) = \frac{1}{2}m(v_{max})^2$$
$$\Rightarrow \quad v_{max} = [2gL(1-\cos\theta)]^{1/2}$$
$$= [2(9.80\text{ m/s}^2)(1.00\text{ m})(1-\cos 15^\circ)]^{1/2}$$
$$= .817\text{ m/s} \quad \text{(and close enough for government work)}$$

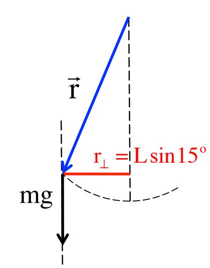


For a_{max}:



Treating the bob as a point mass, we can sum the *torques* about the support:

$$\sum \Gamma_{pin}:$$
$$mg(L\sin\theta) = I\alpha$$
$$\Rightarrow \quad mgL\sin\theta = (mL^2)\alpha$$
$$\Rightarrow \quad \alpha = \frac{g\sin\theta}{L}$$
$$= \frac{(9.80\text{ m/s}^2)\sin 15^\circ}{(1.00\text{ m})}$$
$$= 2.54\text{ rad/s}^2$$



Converting:

$$a_{max} = L\alpha$$
$$= (1.00\text{ m/rad})(2.54\text{ rad/s}^2)$$
$$= 2.54\text{ m/s}^2$$

For F_{max} :

Treating the bob as a point mass, we can determine the force due to gravity at the extreme:

$$F_{max} = mg\sin\theta$$
$$= (.250\ kg)(9.80\ m/s^2)\sin 15^\circ$$
$$= .634\ m/s^2$$